Function Tables: Linear Functions

Topic

Linear functions and tables of their points are examined for patterns.

Problem

What patterns can we find by examining various points on the graph of a linear function, and how we can use these patterns to determine other points on the graph?

AP Association

Functions, Graphs, And Limits

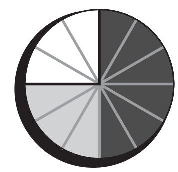
- analysis of graphs

Derivatives

- concept of the derivative
- derivative at a point

Materials

- pencil
- paper
- calculator
- graph paper



Time
30 to 45 minutes

INTRODUCTION

Linear functions are most often studied through coordinate geometry: graphing a line and studying its various geometric attributes, such as slope, *x*-intercept, *y*-intercept, etc. A function also is an “input-output” rule. That is, a number in the domain of the function is put into the function, and its value comes out. These values can be recorded in a table. Sometimes, a table of values is the first information you have about a function. A great deal of information can be found out about a function just by studying a table of its values. In particular, the slope of the linear function takes on an analytical role that is often overlooked when studying the graph of a linear function. Recording the values of a linear function with additively increasing x-values allows you to see the connections between linear functions and arithmetic sequences.

PROCEDURE AND ANALYSIS

In the following, you are given a function and asked to fill in a table of values and investigate the patterns in the table. Conversely, you are given a table of values and asked to find the function that gives the table.

Q1. For each of the given lines, fill in the blanks of the tables (on page 1.01-3).

Describe as many patterns as you can find in each table. Calculate the slope of each linear function. How do the slopes of these functions relate to the patterns that you see?

Q2. The values in each table on page 1.01-4 come from a linear function. Calculate the slope of each linear function. Use the tables from Q1 to derive a method for calculating the slopes that does not involve finding the equations for each line.

Describe your method for finding the slopes.

Q3. Each of the four tables on page 1.01-5 contains points from a line with slope 3. Find the missing numbers. Again, try using the method that does not involve finding the equation of each line.

CONNECTIONS

This module is the first of a series in which the properties of various classes of functions are explored by tables of their values. Next in the series is "Function Tables: Exponential Functions." A surprising amount of similarity between linear functions and exponential functions exists. In "Function

Table 1

Linear functions for problem Q1.

a. $3y + 2x = 1$

x	y
1	
2	
3	
4	
5	

b. $y = \frac{1}{2}x - 1$

x	y
0	
1	
2	
3	
4	

c. $y - 5 = 3(x - 2)$

x	y
0	
.5	
1	
1.5	
2	

d. $y = \frac{3}{4}x + 8$

x	y
0	
.1	
.2	
.3	
.4	

e. $y = -\frac{6}{5}x + \frac{1}{5}$

x	y
$\frac{2}{3}$	
$\frac{5}{6}$	
1	
$\frac{7}{6}$	
$\frac{4}{3}$	

f. $y = 3x + \frac{1}{2}$

x	y
$\frac{1}{2}$	
$\frac{3}{2}$	
$\frac{5}{2}$	
$\frac{7}{2}$	
$\frac{9}{2}$	

Table 2

Function values for problem Q2.

a.

x	y
1	4
2	4.6
3	5.2
4	5.8
5	6.4

b.

x	y
-2	$\frac{18}{4}$
-1	$\frac{29}{6}$
0	$\frac{31}{6}$
1	$\frac{11}{2}$
2	$\frac{35}{6}$

c.

x	y
0.5	1
1.0	.75
1.5	.5

d.

x	y
1	156
11	166
21	176

e.

x	y
0.1	$-\frac{1}{2}$
0.2	$-\frac{1}{4}$
0.3	0

f.

x	y
8	10
15	11

Table 3

Function values for problem Q3.

a.

x	y
5.1	0.38
6.1	
7.1	

b.

x	y
1	10
2	
2.1	
2.11	

c.

x	y
2	0
3	
6	
9	

d.

x	y
	8
	9
4	10
	11
	12

Tables: Polynomial Functions," a slightly different approach is used to find patterns within the function tables of polynomials.

These modules have been written with the intent of encouraging students to analyze patterns. However, if students have a calculator with the ability to find regression curves, they may want to check their results from these problems and in some of the other function tables modules by using the built-in statistics programs. (The Texas Instruments TI-82, TI-83, etc., have this capability.) This is certainly possible and provides a nice connection with some of the students' other classes, especially their science classes.

Although the procedure can be found in the TI-manuals, it is summarized here. First, the students will need to use the STAT key and select the option to EDIT the lists. They then enter the values of the independent variable (x) in one list (L_1 is the usual choice) and the dependent variable (y) in another list (usually this would be L_2). When done entering their data, they need to QUIT this procedure. Finally, they use the STAT CALC menu to select

Typical Answers
Click Icon Above

LinReg(ax+b), or linear regression. The calculator will respond by giving them the curve of the form $y = ax + b$ that most nearly fits the data. For example, in 2e (above) the calculator will give $a = 2.5$ and $b = -.75$, which means that the calculator has selected the linear function $y = 2.5x - .75$. (Students who discover the option to use LinReg(a+bx) should be encouraged to try it.)

Note to teacher: This exercise uses a lot of calculation that will go much more smoothly if the students have access to calculators. And, since this is also a module in problem solving, it is best not to tell the students the correct methods for filling in the tables. Let them work out their methods and strategies. You might be quite surprised in the many ways these problems can be solved.

Q1. Although the computations of these tables may be challenging; the main point of this part of the module is to study the resulting patterns, of which there are quite a few. If the x-values increase additively (that is, by adding a constant to each consecutive term), the function values will do so as well, and the slope of each line will be the same as the function value constant divided by the x-value constant. Also, notice that the function values on each table form an arithmetic sequence.

a. $3y + 2x = 1$

x	y
1	$-0.333\ldots$
2	-1
3	$-1.666\ldots$
4	$-2.333\ldots$
5	-3

b. $y = \frac{1}{2}x - 1$

x	y
0	-1
1	-0.5
2	0
3	0.5
4	1

c. $y - 5 = 3(x - 2)$

x	y
0	-1
.5	.5
1	2
1.5	3.5
2	5

d. $y = \frac{3}{4}x + 8$

x	y
0	
.1	
.2	
.3	
.4	

e. $y = -\frac{6}{5}x + \frac{1}{5}$

x	y
$\frac{2}{3}$	$-\frac{3}{5}$
$\frac{5}{6}$	$-\frac{4}{5}$
1	-1
$\frac{7}{6}$	$-\frac{6}{5}$
$\frac{4}{3}$	$-\frac{7}{5}$

f. $y = 3x + \frac{1}{2}$

x	y
0	$\frac{1}{2}$
$\frac{1}{3}$	$\frac{3}{2}$
$\frac{2}{3}$	$\frac{5}{2}$
1	$\frac{7}{2}$
$\frac{4}{3}$	$\frac{9}{2}$

Q2. Like Q1, there are many different correct methods for finding the slopes of each linear function. In each table, the x-values increase additively by a fixed amount and the y-values increase (or decrease) by a fixed amount as well. The slope will be the y-constant over the x-constant.

a. slope $= 0.6$

b. slope $= \frac{1}{3}$

c. slope $= -\frac{1}{2}$

d. slope $= 1$

e. slope $= -\frac{1}{20}$

f. slope $= \frac{1}{7}$

Q3. Try to get your students to invent methods to solve these without resorting to finding the equations of each line. The key idea to keep in mind is that for each of these tables: $3 = \frac{\Delta y}{\Delta x}$.

a.

x	y
5.1	0.38
6.1	3.38
7.1	6.38

b.

x	y
1	10
2	13
2.1	13.3
2.11	13.33

c.

x	y
2	0
3	3
4	6
5	9

d.

x	y
$\frac{10}{3}$	8
$\frac{11}{3}$	9
4	10
$\frac{13}{3}$	11
$\frac{14}{3}$	12