Function Tables: Exponential Functions

Time
30 to 45 minutes

Topic
Exponential functions and tables of their points are examined for patterns.

Problem
What patterns can we find by examining various points on the graph of an exponential function? How can we use these patterns to determine other points on the graph?

AP Association

FUNCTIONS, GRAPHS, AND LIMITS
- analysis of graphs

DERIVATIVES
- concept of the derivative
- derivative at a point

Materials
- pencil
- paper
- calculator
- graph paper

INTRODUCTION
This module builds on "Function Tables: Linear Functions." In that module, we discovered that certain patterns emerge from discrete data that comes from the graphs of lines. Similar patterns can be found in the discrete data derived from the graphs of exponential functions. Recall that an exponential function is one of the form $f(x) = ka^x$, where k and a are constants. The

constant a is called the base factor of the function f. The importance of this number becomes quite apparent when we study the table of points on an exponential function, just as the slope of a line did with tables of points on a linear function. Recording the values of an exponential function with additively increasing x-values allows you to see the connections between exponential functions and geometric sequences.

Although the exercises in this module largely amount to filling in tables, there is most often more than one way to solve these problems. Calculators of some sort will be needed for some of the computation.

PROCEDURE AND ANALYSIS

Q1. For each of the given lines, fill in the blanks of the chart.

Table 1

Exponential functions for problem Q1.

a. $y = 2^x$

x	y
1	
2	
3	
4	
5	

b. $y = \left(\frac{1}{4}\right)^x$

x	y
0	
1	
2	
3	
4	

c. $y = 5(27)^x$

x	y
0	
$\frac{1}{3}$	
$\frac{2}{3}$	
1	
$\frac{4}{3}$	

d. $y = 3^x$

x	y
0	
.5	
1	
1.5	
2	

e. $y = \left(\frac{4}{9}\right)^x$

x	y
0	
1.5	
3	
4.5	
6	

Describe as many patterns as you can find in each table. How do the base factors of these functions relate to the patterns that you see? How do these patterns compare with those that you found in "Function Tables: Linear Functions?"

Q2. The values in each table come from an exponential function. Calculate the base factor of each function. You may find approximations of each base factor, but try to find exact expressions for each.

Table 2

Exponential values for problem Q2.

a.

x	y
1	2
2	18
3	162
4	1458
5	13122

b.

x	y
−2	.01563
−1	.0625
0	.25
1	1
2	4

c.

x	y
0.5	3
1.0	1.5
1.5	.75

d.

x	y
1	$\frac{1}{3}$
5	$\frac{1}{48}$
9	$\frac{1}{2304}$

e.

x	y
0.1	2
0.2	20
0.3	200

f.

x	y
8	10
15	11

Describe your method for finding the base factors.

Q3. Each of the tables below contains points from an exponential function with base factor 3. Find the missing numbers. Try using a method that does not involve finding the equation of each exponential function.

Table 3

Exponential functions for problem Q3.

a.

x	y
5.1	0.38
6.1	
7.1	

b.

x	y
1	10
2	
2.5	
2.75	

c.

x	y
2	1
3	
9	
27	

d.

x	y
	$\frac{1}{9}$
	1
4	9
	81
	243

CONNECTIONS

There is remarkable similarity between the data tables in this module and those of the "Function Tables: Linear Functions" module. With data tables of linear functions, the y-values will increase additively (i.e., by adding a certain constant) when the x-values increase by additive increments (e.g., going up by 3 each time or down by 5). Furthermore, the slope of the given line will be the ratio of the change in y-values to the change in x-values. With data tables of exponential functions, the y-values will increase multiplicatively (i.e., by multiplying by a certain constant) when the x-values increase by additive increments. The base factor of the given exponential function plays a similar role that the slope played for linear functions.

However, the method of obtaining that number from the data set of values is much more difficult. As we have seen, this method of analyzing functions by looking at discrete sets of data can be quite fruitful.

As with the other function table modules, this module has been written with the intent of encouraging students to analyze patterns. However, if students have a calculator with the ability to find regression curves, they may ask if they can use the built-in statistical packages of their calculators to check their results from problems Q2a–Q2f. (The Texas Instruments TI-82, TI-83, etc., have this capability.) This is certainly possible and provides a nice connection with some of the students' other classes, especially their science classes.

Although the procedure can be found in the TI-manuals, it is summarized here. First, students will need to use the STAT key and select the option to EDIT the lists. They then enter the values of the independent variable (x) in one list (L_1 is the usual choice) and the dependent variable (y) in another list (usually this would be L_2). When done entering their data, they need to QUIT this procedure. Finally, they use the STAT CALC menu to select ExpReg (exponential regression). The calculator will respond by giving them the curve of the form $y = a*b\hat{}x$ that most nearly fits the data. For example, in Q2e (above) the calculator will give $a = .2$ and $b = 1\text{E}10$, which means that the calculator has selected the exponential function $y = .2\,(10^{10})^x$.

Typical Answers
Click Icon Above

Q1. Some of these computations may be a bit challenging, but the main point of this part of the module is to study the resulting patterns. If the x-values increase additively, the function values will increase multiplicatively by a factor that is related to the base factor of the exponential function in question. Also, notice that when the x-values increase additively, the resulting function values form a geometric sequence.

a. $y = 2^x$

x	y
1	2
2	4
3	8
4	16
5	32

b. $y = \left(\frac{1}{4}\right)^x$

x	y
0	1
1	$\frac{1}{4}$
2	$\frac{1}{16}$
3	$\frac{1}{64}$
4	$\frac{1}{256}$

c. $y = 5(27)^x$

x	y
0	5
$\frac{1}{3}$	15
$\frac{2}{3}$	45
1	135
$\frac{4}{3}$	405

d. $y = 3^x$

x	y
0	1
.5	$\sqrt{3} \approx 1.73$
1	3
1.5	$3\sqrt{3} \approx 5.20$
2	9

e. $y = \left(\frac{4}{9}\right)^x$

x	y
0	1
1.5	$\frac{8}{27}$
3	$\frac{8^2}{27^2} = \frac{64}{729}$
4.5	$\frac{8^3}{27^3} = \frac{512}{19683}$
6	$\frac{8^4}{27^4} = \frac{4096}{531441}$

Q2. There are quite a few ways to solve these. Although it is probably the most challenging, the last of these shows the general solution. If the x-values are increasing additively by a constant c while the y-values increase multiplicatively by a factor M, the base factor of the exponential function will be $a = M^{1/c}$.

a. base factor $= 9$
b. base factor $= 4$
c. base factor $= 0.25$
d. base factor $= \frac{1}{2}$
e. base factor $= 10^{10}$
f. base factor $= \left(\frac{11}{10}\right)^{1/7}$

Q3. The key idea is that $3 = (M)^{1/c}$, where M is the multiplicative change in the y-values and c is the additive change in the x-values. Compare this to the third part of the “Function Tables: Linear Functions” module.

a.

x	y
1	10
2	13
2.1	13.3
2.11	13.33

b.

x	y
1	10
2	30
2.5	$30 \times \sqrt{3} \approx 51.96$
2.75	$\left(30 \times \sqrt{3}\right) \times \sqrt[4]{3} \approx 68.39$

c.

x	y
2	1
3	3
4	9
5	27

d.

x	y
0	$\frac{1}{9}$
2	1
4	9
6	81
8	243